Knot theory: an introduction

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USTC, December 25, 2019

Knots in daily life

Shoelace

Knots in daily life

Braids

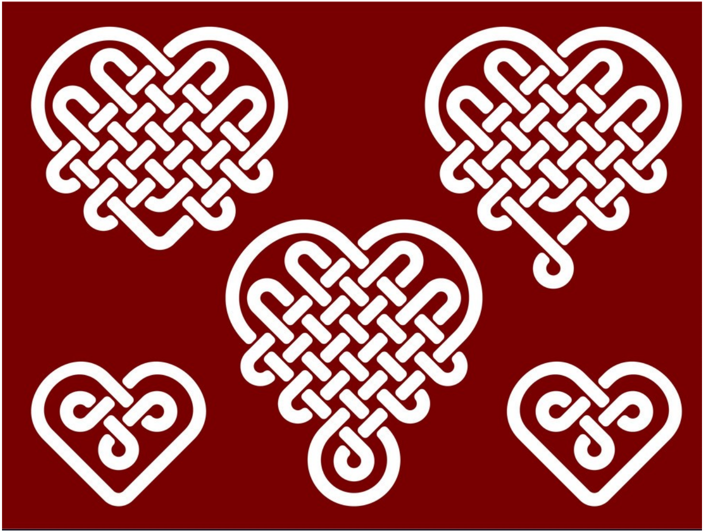
Knot bread


The Knot & Twist
Artisan Pretzels and Breads
Thats How We Roll

German bread: the pretzel

Rope Mat

Chinese Knots



Knots in daily life

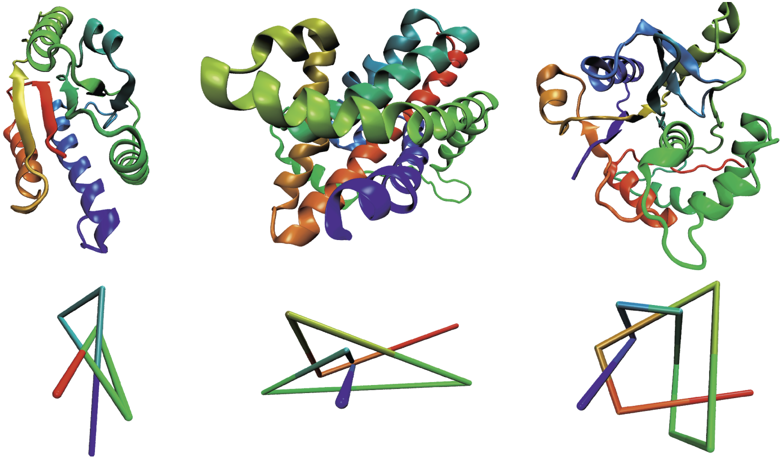
Knot bracelet

Knitting

More Knitting

Knots in daily life

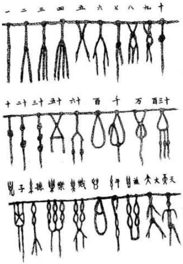
DNA



Knots in daily life

Wire Mess

Chinese talking knots (knotted strings)

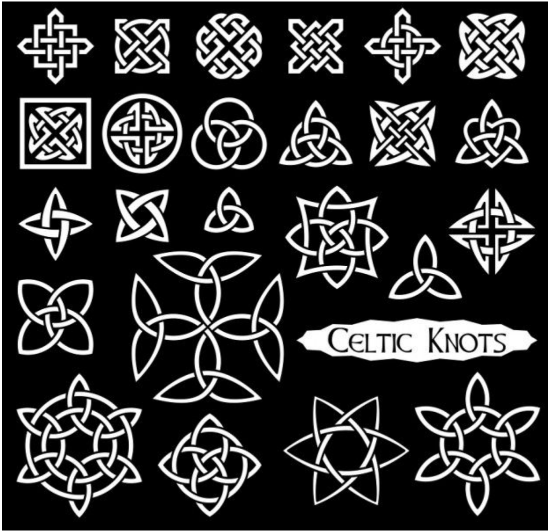


Inca Quipu

Endless Knot in Buddhism

Celtic Knots


Celtic Knots



Basic Questions

- Are knots worthwhile to study?
  Are there any problems on knots worth studying?
- How can knots be viewed as mathematical objects?
  How should we define a knot?
- How should we extract mathematical information from a knot?
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Mathematical Knots

Knots: an embedding of a circle in the three-dimensional world.
Links: an embedding of more than one circles in the three-dimensional world.

Definition: Links and Knots

A link of m components is a subset of S^3 (or $\mathbb{R}^3$) that consists of m disjoint, piecewise linear, simple closed curves.
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Question: Which knots should be considered as "the same"?

Equivalent Links

Links L_1 and L_2 in S^3 are equivalent if there is an orientation-preserving piecewise linear homeomorphism $H : S^3 \longrightarrow S^3$ such that $H(L_1) = L_2$.

Answer: Equivalent links will be regarded as the same link.
In other words, H is isotopic to the identity.

Isotopy

Let g and h be embedding of a link to S^3. An isotopy from g to h is a continuous map $F : S^3 \times [0, 1] \longrightarrow S^3$ for $t \in [0, 1]$ so that F_0 is the identity, each F_t is a homeomorphism, $F_1 \circ g = h$.

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Classification of Knots

Knot theory forms a fundamental source of examples in 3-manifold theory.

A Classical Result

two knots with homeomorphic oriented complements are equivalent.
This is not true generally for links with more than one component.

When we concentrate on knots themselves, start by *mathematicizing* them.

How to represent a knot/link: Link diagram

The image of a link L in $\mathbb{R}^2$ together with "over and under" information at the crossings is called a link diagram of L.

How to judge equivalent knots/links directly: Reidemeister Theorem

Any two diagrams of equivalent links are related by a sequence of Reidemeister moves and an orientation preserving homeomorphisms of the plane.

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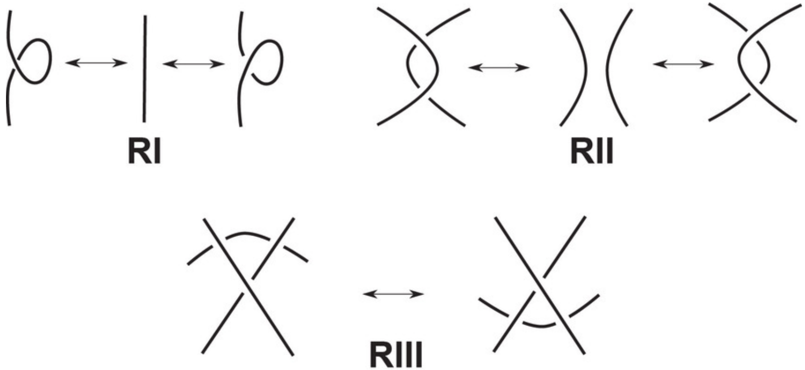
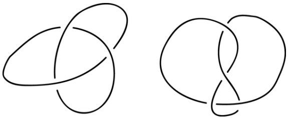
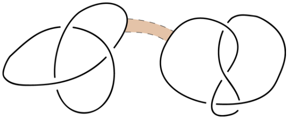
The three types of Reidemeister moves

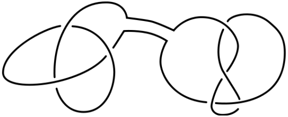




Step I: Start by disjoint planar diagrams.



Step II: Connect by a band with no twist and no extra intersection with the knots.



Step III: Finish by erasing the arcs.

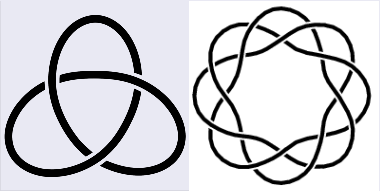
Prime Knots

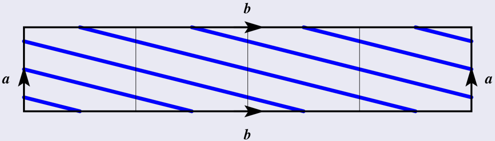
Definition

A knot is a prime knot if it is not the unknot, and $K = K_1 + K_2$ implies that K_1 or K_2 is the unknot.

Example: torus knots

$(p, q)-$torus knot: wrap a circle around a torus p times in one direction and q times in the other, where p and q are coprime integers.





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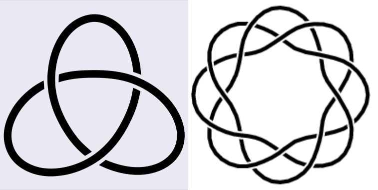
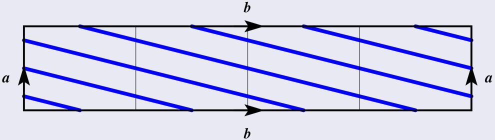
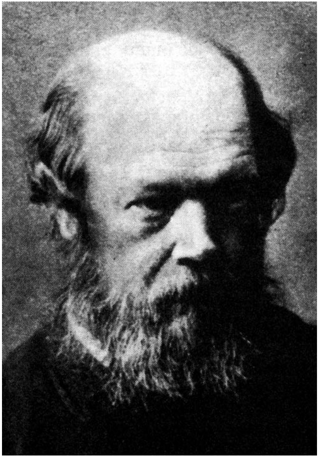
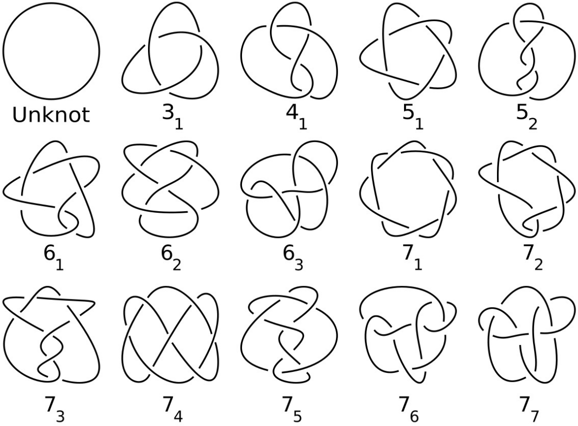




Knot table to Seven Crossings



Peter Guthrie Tait



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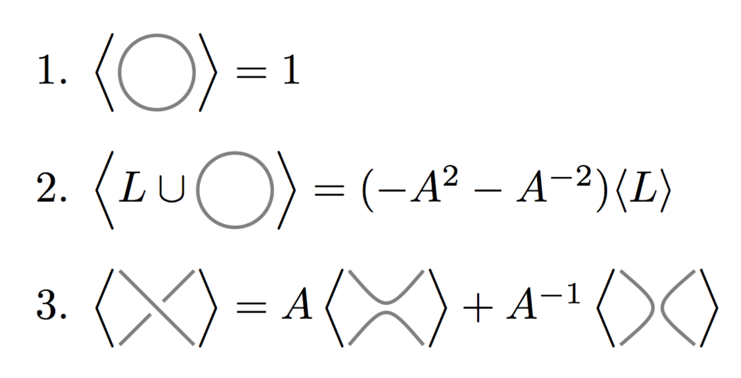
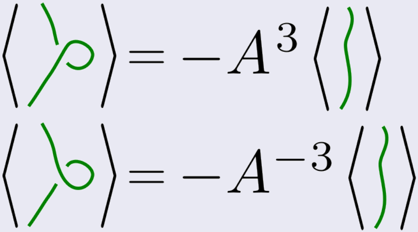
Definition

Unoriented link diagrams $\Rightarrow$ Laurent polynomials with integer coefficients.



Is it a Knot invariant?

Type I Reidemeister Move



Type II and Type III Reidemeister move

$\langle D \rangle$ is invariant.

Kauffman bracket polynomial is NOT an invariant of oriented link.

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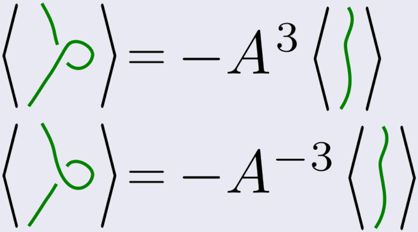

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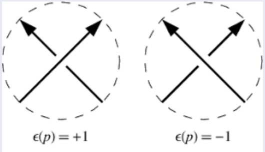
Definition: Writhe

The writhe $w(D)$ of an oriented link D is the sum of the signs of the crossings of D.



Definition: Jones Polynomial $V(L)$ of an oriented link L

D : any oriented diagram of L.

$$V(L) = \left((-A)^{-3w(D)}\langle D\rangle\right)_{t^{\frac{1}{2}}=A^{-2}} \in \mathbb{Z}[t^{-\frac{1}{2}}, t^{\frac{1}{2}}].$$

Jones polynomial is an invariant of oriented links.

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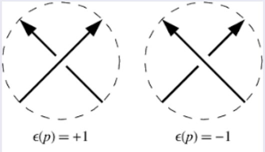

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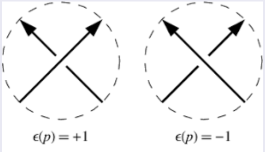

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The Jones polynomial invariant is a function

$$V : \{\text{Oriented links in } S^3\} \longrightarrow \mathbb{Z}[t^{-\frac{1}{2}}, t^{\frac{1}{2}}]$$

such that

1. $V(unknot) = 1$.
2. whenever the three oriented links L_+, L_- and L_0 are the same, except in the neighbourhood of a point where they are as shown in the picture, then we have the skein relation

$$t^{-1}V(L_+) - tV(L_-) + (t^{-\frac{1}{2}} - t^{\frac{1}{2}})V(L_0) = 0.$$

Table 3.1. Jones Polynomial Table

3_1	-1	1	0	1	**0**						
4_1	1	-1	**1**	-1	1						
5_1	-1	1	-1	1	0	1	0	**0**			
5_2	-1	1	-1	2	-1	1	**0**				
6_1	1	-1	1	-2	**2**	-1	1				
6_2	1	-2	2	-2	2	**-1**	1				
6_3	-1	2	-2	**3**	-2	2	-1				
7_1	-1	1	-1	1	-1	1	0	1	0	0	**0**
7_2	-1	1	-1	2	-2	2	-1	1	**0**		
7_3	**0**	0	1	-1	2	-2	3	-2	1	-1	
7_4	**0**	1	-2	3	-2	3	-2	1	-1		
7_5	-1	2	-3	3	-3	3	-1	1	0	**0**	
7_6	-1	2	-3	4	-3	3	**-2**	1			
7_7	-1	3	-3	**4**	-4	3	-2	1			
8_1	1	-1	1	-2	2	-2	**2**	-1	1		
8_2	1	-2	2	-3	3	-2	2	-1	**1**		
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8_4	1	-2	3	-3	3	**-3**	2	-1	1		
8_5	**1**	-1	3	-3	3	-4	3	-2	1		
8_6	1	-2	3	-4	4	-4	3	**-1**	1		
8_7	-1	2	**-2**	4	-4	4	-3	2	-1		
8_8	-1	2	-3	**5**	-4	4	-3	2	-1		
8_9	1	-2	3	-4	**5**	-4	3	-2	1		
8_{10}	-1	2	**-3**	5	-4	5	-4	2	-1		
8_{11}	1	-2	3	-5	5	-4	4	**-2**	1		
8_{12}	1	-2	4	-5	**5**	-5	4	-2	1		
8_{13}	-1	2	-3	5	-5	**5**	-4	3	-1		
8_{14}	1	-3	4	-5	6	-5	4	**-2**	1		
8_{15}	1	-3	4	-6	6	-5	5	-2	1	0	**0**
8_{16}	-1	3	-5	6	-6	6	**-4**	3	-1		
8_{17}	1	-3	5	-6	**7**	-6	5	-3	1		
8_{18}	1	-4	6	-7	**9**	-7	6	-4	1		
8_{19}	**0**	0	0	1	0	1	0	0	-1		
8_{20}	-1	1	-1	2	-1	**2**	-1				
8_{21}	1	-2	2	-3	3	-2	2	**0**			

- oriented link $\Rightarrow$ a Laurent polynomial with integer coefficients.
- an invariant of oriented link
- Alexander polynomial and Jones polynomial give different information of the geometric properties of knots and links.
- Alexander polynomial has longer history than Jones polynomial.
- the construction of Alexander polynomial is based on elementary homology theory.

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The idea of Alexander Polynomial of a Link L

- X: the infinite cyclic cover of the complement of L in S^3.
- Covering transformation acting on X.
- $H_1(X)$: a $\mathbb{Z}[t, t^{-1}]-$module.
  It is called Alexander module.
- the Alexander ideal: an annihilator ideal of the Alexander module.
  It is always principal.
- Alexander polynomial: a generator of the Alexander ideal.

K: any knot. L: any link.

- $\Delta_K(1) = \pm 1$.
- $\Delta_L(t) = \Delta_L(t^{-1})$ up to multiplication by units;
- $\Delta_L(t)$, $\Delta_{mL}(t)$, $\Delta_{rL}(t)$ are equal up to multiplication by units;
- $\Delta_K(t)$ is of the form up to multiplication by units

$$a_0 + a_1(t^{-1} + t) + a_2(t^2 + t^{-2}) + \cdots$$

where a_i are integers and a_0 is odd.

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- $\Delta_L(t)$, $\Delta_{mL}(t)$, $\Delta_{rL}(t)$ are equal up to multiplication by units;
- $\Delta_K(t)$ is of the form up to multiplication by units

$$a_0 + a_1(t^{-1} + t) + a_2(t^2 + t^{-2}) + \cdots$$

where a_i are integers and a_0 is odd.

Features of Alexander polynomial

K: any knot. L: any link.

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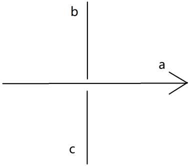
2. Polynomial colouring equations: $a + tc - ta - b = 0$



3. For each crossing, get an equation.
   P_+ :=matrix of coefficients.
4. Delete one row and one column of P_+.
   $P :=$ the resulting matrix.
5. The Alexander polynomial $\Delta_L(t) := \det(P)$.

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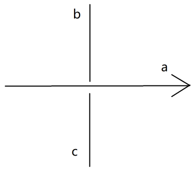

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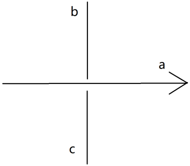

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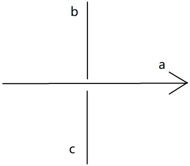

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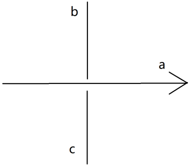

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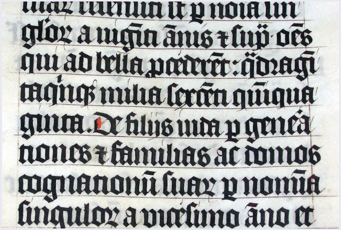
Alexander Polynomial Table

TABLE 6.1. Alexander Polynomial Table

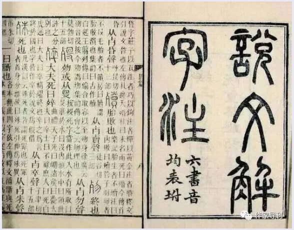
Knot	a_0	a_1	a_2	a_3
3_1	−1	1		
4_1	3	−1		
5_1	1	−1	1	
5_2	−3	2		
6_1	5	−2		
6_2	−3	3	−1	
6_3	5	−3	1	
7_1	−1	1	−1	1
7_2	−5	3		
7_3	3	−3	2	
7_4	−7	4		
7_5	5	−4	2	
7_6	−7	5	−1	
7_7	9	−5	1	
8_1	7	−3		
8_2	3	−3	3	−1
8_3	9	−4		
8_4	−5	5	−2	
8_5	5	−4	3	−1
8_6	−7	6	−2	
8_7	−5	5	−3	1
8_8	9	−6	2	
8_9	7	−5	3	−1
8_{10}	−7	6	−3	1
8_{11}	−9	7	−2	
8_{12}	13	−7	1	
8_{13}	11	−7	2	
8_{14}	−11	8	−2	
8_{15}	11	−8	3	
8_{16}	−9	8	−4	1
8_{17}	11	−8	4	−1
8_{18}	13	−10	5	−1
8_{19}	1	0	−1	1
8_{20}	3	−2	1	
8_{21}	−5	4	−1	

There is Topology in Chinese Gene!

Western words ∼ Permutations and Combinations ∼ Algebra



Chinese Character ∼ Drawing ∼ Topology



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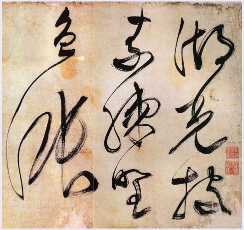
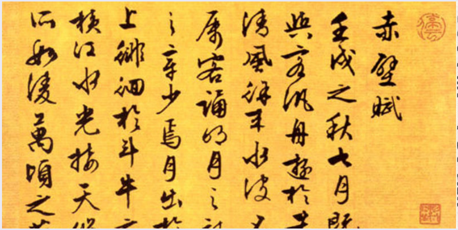
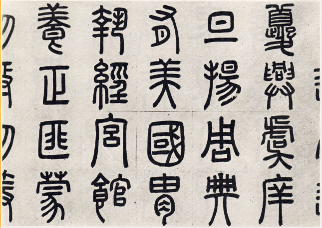
Calligraphy: How can you recognize those words?

草書







Homeomorphisms! Homotopy! Equivalences in Topology!

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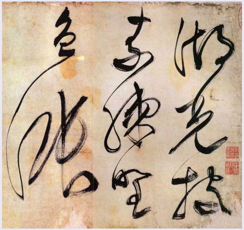

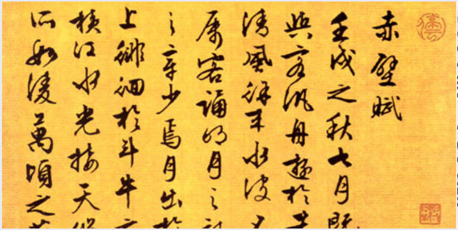

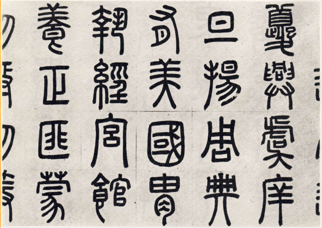

Homeomorphisms! Homotopy! Equivalences in Topology!

There is Topology in Chinese Gene!

Classical Western painting: Present Nature directly in Detail

Classical Chinese painting: Interpret Nature Topologically

Thank you.

Some references

https://huanzhen84.github.io/zhenhuan/HUAN-USTC-2019-KNOTS-Slides.pdf

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